Introduction to Gödels theorem

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Introduction

Any sentence may correspond to a unique integer (sentence number) and any integer may correspond to a unique sentence. This can be done by allocating an integer to every symbol in the sentences of interest. As an example, the integers 1-52 may correspond to the letters A - z, if these are the only symbols in the sentences. In general sentences also contain other symbols.
The sentence number is now calculated by the formula:

$$s = 2^{n1}3^{n2}5^{n3}7^{n4} \; . \; . \; . \; . \; pl^{nl}$$

where ni is the integer for the i'th symbol in the sentence and pi is the i'th prime number. The number of symbols in the sentence is l.
See D. Hofstader: Limits of Logic: The Gödel Legacy
There is thus an isomorphism between the integers > 1 and the set of sentences, since the prime factoring is unique.
If a sentence is well formed (without self reference) and can be proven true, it is called a theorem. The sentence number is then called a theorem number.
An example of a self referencing sentence is:

This sentence is false

and cannot be a theorem.

Definition of the number g

The sentence:

The number g is not a theorem number

has a sentence number that is easily computed. This number is called g. In the following g refers to this specific number.

An example of a theorem

We start with the sentence:

The number g is not a prime number

This is a well formed sentence, it is not self referencing and it is either true or false. This can be determined by checking the number g. It might therefore be a theorem.

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We now turn to what it's all about, the sentence:

The number g is not a theorem number

where g refers to the number above.
This is a well formed sentence, it is not self referencing and is, like the sentence in the example, just a statement about a number and might therefore be a theorem. This can be determined by checking whether the sentence is true or false.
We assume that the sentence is false, that is g is a theorem number. If g is a theorem number, then the sentence is true, as the sentence is described by the sentence number g.
We assume that the sentence is true, that is g is not a theorem number. If g is not a theorem number, then the sentence is false, as the sentence is described by the sentence number g.
It is therefore not possible to say whether the sentence "The number g is not a theorem umber" is true or false. One can say that Gödel has smuggeled self reference in through a backdor like a trojan horse by making an isomorphism between sentences and integers. The sentence "The number g is not a theorem number" only expresses something about a number and is not self referencing, but as the number g is a sentence number, we are in a way dealing with self referencing. Gödels theory is much more than described here and contains two theorems.

Commentary

It is thus not true, as some people claim, that Gödels theorems is about the existence of true sentences (theorems) that cannot be proven.