Design and properties of the Moriat window

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Abstract

A new class of optimal window functions, the so called Moriat windows (developed by the legendary french engineer Alain Moriat) is presented. The window class is a subblass of the widely used cosine window class. The Moriat windows are optimal in the sense that the windows are zero and maximally flat at each end, that is, the first 2P-1 derivatives of the P'th order window are zero, where P is the nunber of cosines.
The windows are also optimal in the sense that the sidelobes of the frequency response of the windows have a maximal roll off. Apart from beeing optimal the window coefficients are given in closed form and are very easy to calculate. A special case of the Moriat windows is the 1'st order window, which is identical to the well known Hanning window.

Definition

The P'th order cosine window can be expressed as:

$$w(i) = 1 + \sum_{n=1}^{P} \beta_P(n) \cos(\frac{2\pi}{N} ni) \qquad\qquad i = 0, ..., N-1$$

And the frequency response is given by:

$$W(\omega) = \frac{\sin(\pi\omega)}{\pi\omega}(1 + \omega^2 \sum_{n=1}^{P} \frac{\beta_P(n)}{\omega^2 - n^2})$$

For the Moriat window the following conditions must be satisfied:

$$\sum_{n=1}^{P} \beta_P(n) = -1$$

and

$$\sum_{n=1}^{P} n^{2P-2} \beta_P(n) = 0$$

As a consequense the window can be expressed as:

$$w(i) = \cos^{2P}(\frac{\pi}{N}(i - \frac{N}{2} + 1))$$

Window coefficients

It is easy to show that these conditions lead to the following expression for the frequency response:

$$W(\omega) = \frac{\sin(\pi\omega)}{\pi\omega} \frac{(P!)^2}{\prod_{n=1}^{P}(n^2 - \omega^2)}$$

From this and the general equation for the frequency response it is easy to derive the window coefficients:

$$\beta_P(n) = -\prod_{l=1}^{P} \frac{l^2}{l^2 - n^2} = (-1)^n \frac{2(P!)^2}{(P-n)!(P+n)!}$$

Properties

When using windows for sine wave analysis it is often desired that one can find the exact frequency of the sine wave from the two highest lines in the frequency response. In general it is not possible to find an exact solution to this problem, but for the Moriat windows the relation is given by:

$$\Delta\omega = \frac{1 + P(1 - G)}{1 + G}$$

where:
G is the level ratio between the highest and the second higher line in the frequency response.
$\Delta\omega$ is the difference between the exact frequency and the frequency of the highest line in the frequency response.

Examples

The coefficients of the first four Moriat windows are given by:

Order	Coefficients
$\beta_1(1)$	-1
$\beta_2(1)$	-4/3
$\beta_2(2)$	1/3
$\beta_3(1)$	-3/2
$\beta_3(2)$	3/5
$\beta_3(3)$	-1/10
$\beta_4(1)$	-8/5
$\beta_4(2)$	4/5
$\beta_4(3)$	-8/35
$\beta_4(4)$	1/35